Appendix A: Algebra Review

SECTION A.1: Algebraic Expressions

1. Undefined terms, definitions, axioms or postulates, theorems

2. Algebra; geometry

3. a. Reflexive
   b. Transitive
   c. Substitution
   d. Symmetric

4. a. $2 = 2$
   b. If $2 = x$, then $x = 2$.
   c. If $x = 2$ and $2 = y$, then $x = y$.
   d. If $2 + 5 = 7$ and $7 + y = z$, then $2 + 5 + y = z$.

5. a. 12
   b. –2
   c. 2
   d. –12

6. a. 8
   b. –8
   c. –22
   d. 1

7. a. 35
   b. –35
   c. –35
   d. 35

8. a. –84
   b. 84
   c. –84
   d. 84

9. No; Commutative Axiom for Multiplication

10. a. Commutative Axiom for Multiplication
    b. Associative Axiom for Addition
    c. Commutative Axiom for Addition
    d. Associative Axiom for Multiplication

11. a. 9
    b. –9
    c. 8
    d. –8

12. $(-3) - 7$

13. a. –4
    b. –36
    c. 18
    d. $-\dfrac{1}{4}$

14. 5 feet divided by 10 spaces = $\dfrac{1}{2}$ ft per space. Or since 5 feet = 60 inches, 60 inches divided by 10 spaces = 6 inches per space.

15. –$60

16. $25(2) + 30(2) = 50 + 60 = \$110$

17. a. $30 + 35 = 65$
    b. $28 - 12 = 16$
    c. $\dfrac{7}{2} + \dfrac{11}{2} = \dfrac{18}{2} = 9$
    d. $8x$

18. a. $54 - 24 = 30$
    b. $3(4 + 8) = 12 + 24 = 36$
    c. $7y - 2y = (7 - 2)y = 5y$
    d. $(16 + 8)x = 24x$

19. a. $(6 + 4)\pi = 10\pi$
    b. $(8 + 3)\sqrt{2} = 11\sqrt{2}$
    c. $(16 - 9)x^2 y = 7x^2 y$
    d. $(9 - 2)\sqrt{3} = 7\sqrt{3}$

20. a. $(1 + 2)\pi r^2 = 3\pi r^2$
    b. $(7 + 3)xy = 10xy$
    c. $7x^2 y + 3xy^2$
    d. $(1 + 1)x + y = 2x + y$

21. **a.** $2 + 12 = 14$

  b. $5 \cdot 4 = 20$

  c. $2 + 3 \cdot 4 = 2 + 12 = 14$

  d. $2 + 6^2 = 2 + 36 = 38$

22. **a.** $9 + 16 = 25$

  b. $7^2 = 49$

  c. $9 + 6 \div 3 = 9 + 2 = 11$

  d. $[9 + 6] \div 3 = 15 \div 3 = 5$

23. **a.** $\dfrac{6}{-6} = -1$

  b. $\dfrac{8 - 6}{6 \cdot 3} = \dfrac{2}{18} = \dfrac{1}{9}$

  c. $\dfrac{10 - 18}{9} = \dfrac{-8}{9}$

  d. $\dfrac{5 - 12 + (-3)}{4 + 16} = \dfrac{-10}{20} = \dfrac{-1}{2}$

24. **a.** $8 + 10 + 12 + 15 = 45$

  b. $42 + 7 - 12 - 2 = 35$

25. **a.** $15 - 6 - 5 + 2 = 6$

  b. $12x^2 - 15x + 8x - 10 = 12x^2 - 7x - 10$

26. **a.** $10x^2 - 35x + 6x - 21 = 10x^2 - 39x - 21$

  b. $6x^2 - 10xy + 3xy - 5y^2 = 6x^2 - 7xy - 5y^2$

27. $5x + 2y$

28. $2xy + 2yz + 2xz$

29. $10x + 5y$

30. $xy + xz + y^2 + yz$; the total of the areas of the four smaller plots is also $xy + y^2 + xz + yz$.

31. $10x$

32. $9\pi + 48\pi + 9\pi = 66\pi$

SECTION A.2: Formulas and Equations

1. $5x + 8$

2. $-1x + 8$

3. $2x - 2$

4. $4x - 2$

5. $2x + 2 + 3x + 6 = 5x + 8$

6. $6x + 15 - 6x + 2 = 17$

7. $x^2 + 4x + 3x + 12 = x^2 + 7x + 12$

8. $x^2 - 7x - 5x + 35 = x^2 - 12x + 35$

9. $6x^2 - 4x + 5x - 10 = 6x^2 + 11x - 10$

10. $6x^2 + 9x + 14x + 21 = 6x^2 + 23x + 21$

11. $(a + b)(a + b) + (a - b)(a - b)$
$= a^2 + ab + ab + b^2 + a^2 - ab - ab + b^2$
$= 2a^2 + 2b^2$

12. $(x + 2)(x + 2) - (x - 2)(x - 2)$
$= x^2 + 2x + 2x + 4 - \left(x^2 - 2x - 2x + 4\right)$
$= x^2 + 4x + 4 - \left(x^2 - 4x + 4\right)$
$= x^2 + 4x + 4 - x^2 + 4x - 4$
$= 8x$

13. $4 \cdot 3 \cdot 5 = 60$

14. $5^2 + 7^2 = 35 + 49 = 74$

15. $2 \cdot 13 + 2 \cdot 7 = 26 + 14 = 40$

16. $6 \cdot 16 \div 4 = 96 \div 4 = 24$

17. $S = 2 \cdot 6 \cdot 4 + 2 \cdot 4 \cdot 5 + 2 \cdot 6 \cdot 5$
$S = 48 + 40 + 60$
$S = 148$

18. $A = \left(\dfrac{1}{2}\right) 2(6 + 8 + 10)$
$A = 1(24)$
$A = 24$

19. $V = \left(\dfrac{1}{3}\right) \pi (3)^2 \cdot 4$
$V = \dfrac{1}{3} \cdot \pi \cdot 9 \cdot 4$
$V = 12\pi$

20. $S = 4\pi r^2$
$S = 4\pi r(2)^2$
$S = 4\pi \cdot 4$
$S = 16\pi$

21. $2x = 14$
   $x = 7$

22. $3x = -3$
   $x = -1$

23. $\dfrac{y}{-3} = 4$
   $y = -12$

24. $7y = -21$
   $y = -3$

25. $2a + 2 = 26$
   $2a = 24$
   $a = 12$

26. $\dfrac{3b}{2} = 27$
   $3b = 54$
   $b = 18$

27. $2x + 2 = 30 - 6x + 12$
   $2x + 2 = 42 - 6x$
   $8x = 40$
   $x = 5$

28. $2x + 2 + 3x + 6 = 22 + 40 - 4x$
   $5x + 8 = 62 - 4x$
   $9x = 54$
   $x = 6$

29. Multiply equation by 6 to get
   $2x - 3x = -30$
   $-x = -30$
   $x = 30$

30. Multiply equation by 12 to get
   $6x + 4x + 3x = 312$
   $13x = 312$
   $x = 24$

31. Multiply equation by n to get
   $360 + 135n = 180n$
   $360 = 45n$
   $8 = n$

32. Multiply equation by n to get
   $(n - 2)180 = 150n$
   $180n - 360 = 150n$
   $-360 = -30n$
   $12 = n$

33. $148 = 2 \cdot 5 \cdot w + 2 \cdot w \cdot 6 + 2 \cdot 5 \cdot 6$
   $148 = 10w + 12w + 60$
   $88 = 22w$
   $4 = w$

34. $156 = \left(\dfrac{1}{2}\right) \cdot 12 \cdot (b + 11)$
   $156 = 6(b + 11)$
   $156 = 6b + 66$
   $90 = 6b$
   $15 = b$

35. $23 = \left(\dfrac{1}{2}\right)(78 - y)$
   $46 = 78 - y$
   $-32 = -y$
   $32 = y$

36. $\dfrac{-3}{2} = \dfrac{Y - 1}{2 - (-2)}$
   $\dfrac{-3}{2} = \dfrac{Y - 1}{4}$
   $2Y - 2 = -12$
   $2Y = -10$
   $Y = -5$

SECTION A.3: Inequalities

1. The length of $\overline{AB}$ is greater than the length of $\overline{CD}$.

2. $e < f;\ f > e$

3. The measure of angle ABC is greater than the measure of angle DEF.

4. $x = 6,\ x = 9,\ x = 12$

5. **a.** $p = 4$

   b. $p = 10$

6. Yes

7. **a.** $a = -6,\ b = -8$
       $a > b$

   b. $a = 8,\ b = 9$
       $a < b$

8. **a.** $a = 1,\ b = 1$
       $a = b$

   b. $a = 10,\ b = 14$
       $a < b$

9. $AB > IJ$

10. The measure of angle JKL is greater than the measure of angle ABC.

11. **a.** False

   b. True

   c. True

   d. False

12. **a.** True

  b. False

  c. True

 **d.** False

13. The measure of the second angle must be greater than 148° and less than 180°.

14. The length of the second board must be less than 5 feet.

15. **a.** $-12 \le 20$

  b. $-10 \le -2$

  c. $18 \ge -30$

  d. $3 \ge -5$

16. **a.** $2 > -1$

  b. $12 < 18$

  c. $-12 > -18$

  d. $2 < 3$

17.

No Change	No Change
No Change	No Change
No Change	CHANGE
No Change	CHANGE

18. $5x \le 30$
  $x \le 6$
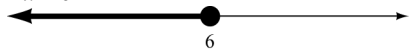
19. $2x \le 14$
  $x \le 7$
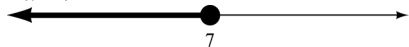
20. $4x > 20$
  $x > 5$
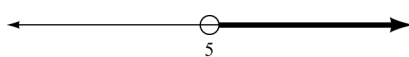
21. $-4x > 20$
  $x < -5$
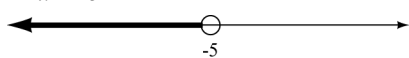
22. $10 - 5x \le 30$
  $-5x \le 20$
  $x \ge -4$

23. $5x < 200 - 5x$
  $10x < 200$
  $x < 20$
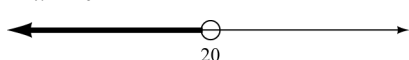
24. $5x + 10 < 54 - 6x$
  $11x < 44$
  $x < 4$
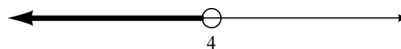
25. $2x - 3x \le 24$
  $-x \le 24$
  $x \ge -24$
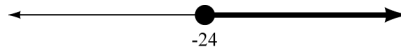
26. $2x - 3 < -35$
  $2x < -32$
  $x < -16$
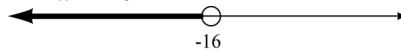
27. $x^2 + 4x \le x^2 - 5x - 18$
  $9x \le -18$
  $x \le -2$
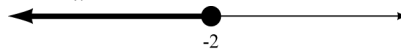
28. $x^2 + 2x < 2x - x^2 + 2x^2$
  $x^2 + 2x < x^2 + 2x$
  No solution or $\varnothing$.

29. Not true if $c < 0$.

30. Not true if $c = 0$.

31. Not true if $a = -3$ and $b = -2$.

32. Not true if $a = c$.

SECTION A.4: Factoring and Quadratic Equations

1. $a(x^2 + 5x + 7)$

2. $5y^2(y - 4)$

3. $2bx(x + 2b)$

4. $4(x + 3y + 2z)$

5. $(y + 3)\,(y - 3)$

6. $(4x + 3y)\,(4x - 3y)$

7. $(2x + 7y)\,(2x - 7y)$

8. $(a + 10)\,(a - 10)$

9. $(x + 3)\,(x + 4)$

10. $(x - 7)\,(x - 2)$

11. $(x + 8)\,(x - 3)$

12. $(y - 12)\,(y + 8)$

13. $(2y + 3)\,(3y - 2)$

14. $(4a + 5)\,(3a + 4)$

15. $(3x - y)\,(x + 4y)$

16. $(2a + 3b)\,(2a + 3b)$

17. $4(x + 2)\,(x - 2)$

18. $6(y + 3)\,(y - 3)$

19. $3(y + 3)\,(y + 5)$

20. $5(3x - 2)\,(2x - 1)$

21. $a(2x - 7)\,(x + 5)$

22. $c^2(3a - 2b)\,(2a + 5b)$

23. $x(x + 4)\,(x + 1)$

24. $x(x + 3)\,(x - 3)$

25. $x^2 - 6x + 8 = 0$
$(x - 4)(x - 2) = 0$
$x - 4 = 0 \quad \text{or} \quad x - 2 = 0$
$x = 4 \quad \text{or} \qquad x = 2$

26. $x^2 + 4x = 21$
$x^2 + 4x - 21 = 0$
$(x + 7)(x - 3) = 0$
$x + 7 = 0 \quad \text{or} \quad x - 3 = 0$
$x = -7 \quad \text{or} \qquad x = 3$

27. $3x^2 - 51x + 180 = 0$
$3(x^2 - 17x + 60) = 0$
$3(x - 12)(x - 5) = 0$
$x - 12 = 0 \quad \text{or} \quad x - 5 = 0$
$x = 12 \quad \text{or} \qquad x = 5$

28. $2x^2 + x - 6 = 0$
$(2x - 3)(x + 2) = 0$
$2x - 3 = 0 \quad \text{or} \quad x + 2 = 0$
$2x = 3 \quad \text{or} \qquad x = -2$
$x = \dfrac{3}{2} \quad \text{or} \qquad x = -2$

29. $3x^2 = 10x + 8$
$3x^2 - 10x - 8 = 0$
$(3x + 2)(x - 4) = 0$
$3x + 2 = 0 \quad \text{or} \quad x - 4 = 0$
$3x = -2 \quad \text{or} \qquad x = 4$
$x = -\dfrac{2}{3} \quad \text{or} \qquad x = 4$

30. $8x^2 + 40x - 112 = 0$
$8(x^2 + 5x - 14) = 0$
$8(x + 7)(x - 2) = 0$
$x + 7 = 0 \quad \text{or} \quad x - 2 = 0$
$x = -7 \quad \text{or} \qquad x = 2$

31. $6x^2 = 5x - 1$
$6x^2 - 5x + 1 = 0$
$(3x - 1)(2x - 1) = 0$
$3x - 1 = 0 \quad \text{or} \quad 2x - 1 = 0$
$3x = 1 \quad \text{or} \qquad 2x = 1$
$x = \dfrac{1}{3} \quad \text{or} \qquad x = \dfrac{1}{2}$

32. $12x^2 + 10x = 12$
$12x^2 + 10x - 12 = 0$
$2(6x^2 + 5x - 6) = 0$
$2(3x - 2)(2x + 3) = 0$
$3x - 2 = 0 \quad \text{or} \quad 2x + 3 = 0$
$3x = 2 \quad \text{or} \qquad 2x = -3$
$x = \dfrac{2}{3} \quad \text{or} \qquad x = -\dfrac{3}{2}$

33. $2x(x - 3) = 0$
$2x = 0 \quad \text{or} \quad x - 3 = 0$
$x = 0 \quad \text{or} \qquad x = 3$

34. $(4x - 3)(4x + 3) = 0$
$4x - 3 = 0 \quad \text{or} \quad 4x + 3 = 0$
$4x = 3 \quad \text{or} \qquad 4x = -3$
$x = \dfrac{3}{4} \quad \text{or} \qquad x = \dfrac{-3}{4}$

35. $4y^2 = 25$
$$4y^2 - 25 = 0$$
$$(2y + 5)(2y - 5) = 0$$
$2y + 5 = 0$ or $2y - 5 = 0$
$2y = -5$ or $2y = 5$
$y = \dfrac{-5}{2}$ or $y = \dfrac{5}{2}$

36. $9y^2 - 18y = 0$
$9y(y - 2) = 0$
$9y = 0$ or $y - 2 = 0$
$y = 0$ or $y = 2$

37. $w^2 + 5w - 66 = 0$
$(w + 11)(w - 6) = 0$
$w + 11 = 0$ or $w - 6 = 0$
$w = -11$ or $w = 6$ reject $w = -11$
width = 6; length = 11

38. $x^2 + 5x - 24 = 0$
$(x + 8)(x - 3) = 0$
$x + 8 = 0$ or $x - 3 = 0$
$x = -8$ or $x = 3$
reject $x = -8$; $x = 3$

39. $a^2 + a^2 + 2a + 1 - 25 = 0$
$$2a^2 + 2a - 24 = 0$$
$$2(a^2 + a - 12) = 0$$
$$2(a + 4)(a - 3) = 0$$
$a + 4 = 0$ or $a - 3 = 0$
$a = -4$ or $a = 3$
reject $a = -4$; $a = 3$

40. $4x^3 - 28x^2 + 24x = 0$
$4x(x^2 - 7x + 6) = 0$
$4x\ (x - 6)(x - 1) = 0$
$4x = 0$ or $x - 6 = 0$ or $x - 1 = 0$
$x = 0$ or $x = 6$ or $x = 1$

41. $(x^2 - 9)(x^2 - 4) = 0$
$(x - 3)(x + 3)(x - 2)(x + 2) = 0$
$x - 3 = 0$ or $x + 3 = 0$ or
$x - 2 = 0$ or $x + 2 = 0$
$x = 3$ or $x = -3$ or
$x = 2$ or $x = -2$

SECTION A.5: The Quadratic Formula and Square Root Properties

1. **a.** 3.61

   b. 2.83

   c. –5.39

   d. 0.77

2. **a.** 4.12

   b. 20

   c. –2.65

   d. 1.26

3. a, c, d, f

4. a, b, c, e

5. **a.** $\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

   b. $\sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$

   c. $\sqrt{900} = 30$

   d. $\left(\sqrt{3}\right)^2 = 3$

6. **a.** $\sqrt{28} = \sqrt{4 \cdot 7} = 2\sqrt{7}$

   b. $\sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$

   c. $\sqrt{54} = \sqrt{9 \cdot 6} = 3\sqrt{6}$

   d. $\sqrt{200} = \sqrt{100 \cdot 2} = 10\sqrt{2}$

7. **a.** $\sqrt{\dfrac{9}{16}} = \dfrac{\sqrt{9}}{\sqrt{16}} = \dfrac{3}{4}$

   b. $\sqrt{\dfrac{25}{49}} = \dfrac{\sqrt{25}}{\sqrt{49}} = \dfrac{5}{7}$

   c. $\sqrt{\dfrac{7}{16}} = \dfrac{\sqrt{7}}{\sqrt{16}} = \dfrac{\sqrt{7}}{4}$

   d. $\sqrt{\dfrac{6}{9}} = \dfrac{\sqrt{6}}{\sqrt{9}} = \dfrac{\sqrt{6}}{3}$

8. **a.** $\sqrt{\dfrac{1}{4}} = \dfrac{\sqrt{1}}{\sqrt{4}} = \dfrac{1}{2}$

   b. $\sqrt{\dfrac{16}{9}} = \dfrac{\sqrt{16}}{\sqrt{9}} = \dfrac{4}{3}$

   c. $\sqrt{\dfrac{5}{36}} = \dfrac{\sqrt{5}}{\sqrt{36}} = \dfrac{\sqrt{5}}{6}$

   d. $\sqrt{\dfrac{3}{16}} = \dfrac{\sqrt{3}}{\sqrt{16}} = \dfrac{\sqrt{3}}{4}$

9. a. $\sqrt{54} \approx 7.35$ and $3\sqrt{6} \approx 7.35$

b. $\sqrt{\dfrac{5}{16}} \approx 0.56$ and $\dfrac{\sqrt{5}}{4} \approx 0.56$

10. a. $\sqrt{48} \approx 6.93$ and $4\sqrt{3} \approx 6.93$

b. $\sqrt{\dfrac{7}{9}} \approx 0.88$ and $\dfrac{\sqrt{7}}{3} \approx 0.88$

11. $a = 1, b = -6, c = 8$

12. $a = 2, b = -1, c = -3$

13. $a = 1, b = -4, c = -12$

14. $a = 1, b = 6, c = -40$

15. $a = 3, b = -10, c = -25$

16. $a = 5, b = 2, c = -90$

17. $a = 2, b = 3, c = -152$

18. $a = 15, b = -2, c = -56$

19. $x^2 - 7x + 10 = 0$
$a = 1, \ b = -7, \ c = 10$
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \dfrac{7 \pm \sqrt{49 - 4(1)(10)}}{2(1)}$
$x = \dfrac{7 \pm \sqrt{49 - 40}}{2}$
$x = \dfrac{7 \pm \sqrt{9}}{2}$
$x = \dfrac{7 + 3}{2}$ or $x = \dfrac{7 - 3}{2}$
$x = 5$ or 2

20. $x^2 + 7x + 12 = 0$
$a = 1, \ b = 7, \ c = 12$
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \dfrac{-7 \pm \sqrt{49 - 4(1)(12)}}{2(1)}$
$x = \dfrac{-7 \pm \sqrt{49 - 48}}{2}$
$x = \dfrac{-7 \pm \sqrt{1}}{2}$
$x = \dfrac{-7 + 1}{2}$ or $x = \dfrac{-7 - 1}{2}$
$x = -3$ or -4

21. $x^2 + 9 = 7x$
$x^2 - 7x + 9 = 0$
$a = 1, \ b = -7, \ c = 9$
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \dfrac{7 \pm \sqrt{49 - 4(1)(9)}}{2(1)}$
$x = \dfrac{7 \pm \sqrt{49 - 36}}{2}$
$x = \dfrac{7 \pm \sqrt{13}}{2} \approx 5.30$ or 1.70

22. $2x^2 + 3x = 6$
$2x^2 + 3x - 6 = 0$
$a = 2, \ b = 3, \ c = -6$
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \dfrac{-3 \pm \sqrt{9 - 4(2)(-6)}}{2(2)}$
$x = \dfrac{-3 \pm \sqrt{9 + 48}}{4}$
$x = \dfrac{-3 \pm \sqrt{57}}{4} \approx 1.14$ or -2.64

23. $x^2 - 4x - 8 = 0$
$a = 1, \ b = -4, \ c = -8$
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \dfrac{4 \pm \sqrt{16 - 4(1)(-8)}}{2(1)}$
$x = \dfrac{4 \pm \sqrt{16 + 32}}{2}$
$x = \dfrac{4 \pm \sqrt{48}}{2}$
$x = \dfrac{4 \pm 4\sqrt{3}}{2} \approx 5.46$ or -1.46

24. $x^2 - 6x - 2 = 0$
$a = 1, \ b = -6, \ c = -2$
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \dfrac{6 \pm \sqrt{36 - 4(1)(-2)}}{2(1)}$
$x = \dfrac{6 \pm \sqrt{36 + 8}}{2}$
$x = \dfrac{6 \pm \sqrt{44}}{2}$
$x = \dfrac{6 \pm 2\sqrt{11}}{2}$
$x = 3 \pm \sqrt{11} \approx 6.32$ or -0.32

25.
$$5x^2 = 3x + 7$$
$$5x^2 - 3x - 7 = 0$$
$$a = 5, \quad b = -3, \quad c = -7$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{3 \pm \sqrt{9 - 4(5)(-7)}}{2(5)}$$
$$x = \frac{3 \pm \sqrt{9 + 140}}{10}$$
$$x = \frac{3 \pm \sqrt{149}}{10} \approx 1.52 \text{ or } -0.92$$

26.
$$2x^2 = 8x - 1$$
$$2x^2 - 8x + 1 = 0$$
$$a = 2, \quad b = -8, \quad c = 1$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{8 \pm \sqrt{64 - 4(2)(1)}}{2(2)}$$
$$x = \frac{8 \pm \sqrt{64 - 8}}{4}$$
$$x = \frac{8 \pm \sqrt{56}}{4}$$
$$x = \frac{8 \pm 2\sqrt{14}}{4}$$
$$x = \frac{4 \pm \sqrt{14}}{2} \approx 3.87 \text{ or } 0.13$$

27.
$$2x^2 = 14$$
$$x^2 = 7$$
$$x = \pm\sqrt{7}$$
$$x \approx \pm 2.65$$

28.
$$2x^2 = 14x$$
$$2x^2 - 14x = 0$$
$$2x(x - 7) = 0$$
$$2x = 0 \quad \text{or} \quad x - 7 = 0$$
$$x = 0 \quad \text{or} \quad x = 7$$

29.
$$4x^2 - 25 = 0$$
$$4x^2 = 25$$
$$x^2 = \frac{25}{4}$$
$$x = \pm\frac{5}{2}$$

30.
$$4x^2 - 25x = 0$$
$$x(4x - 25) = 0$$
$$x = 0 \quad \text{or} \quad 4x - 25 = 0$$
$$x = 0 \quad \text{or} \quad 4x = 25$$
$$x = 0 \quad \text{or} \quad x = \frac{25}{4}$$

31.
$$ax^2 - bx = 0$$
$$x(ax - b) = 0$$
$$x = 0 \quad \text{or} \quad ax - b = 0$$
$$x = 0 \quad \text{or} \quad ax = b$$
$$x = 0 \quad \text{or} \quad x = \frac{b}{a}$$

32.
$$ax^2 - b = 0$$
$$ax^2 = b$$
$$x^2 = \frac{b}{a}$$
$$x = \pm\sqrt{\frac{b}{a}}$$
$$x = \pm\frac{\sqrt{ab}}{a}$$

33. Let the length = x + 3 and width = x. The area is then:
$$x(x + 3) = 40$$
$$x^2 + 3x = 40$$
$$x^2 + 3x - 40 = 0$$
$$(x + 8)(x - 5) = 0$$
$$x + 8 = 0 \quad \text{or} \quad x - 5 = 0$$
$$x = -8 \quad \text{or} \quad x = 5$$

Reject x = −8 because the length cannot be negative. The rectangle is 5 by 8.

34.
$$x \cdot (x + 5) = (x + 1) \cdot 4$$
$$x^2 + 5x = 4x + 4$$
$$x^2 + 1x - 4 = 0$$
$$a = 1, \quad b = 1, \quad c = -4$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-1 \pm \sqrt{1 - 4(1)(-4)}}{2(1)}$$
$$x = \frac{-1 \pm \sqrt{1 + 16}}{2}$$
$$x = \frac{-1 \pm \sqrt{17}}{2}$$
$$CP = \frac{-1 + \sqrt{17}}{2} \approx 1.56$$

$$\frac{-1 - \sqrt{17}}{2} \text{ is rejected because it is a negative number.}$$

35.
$$D = \frac{n(n - 3)}{2}$$
$$9 = \frac{n(n - 3)}{2}$$
$$18 = n^2 - 3n$$
$$0 = n^2 - 3n - 18$$
$$0 = (n - 6)(n - 3)$$
$$n = 6 \text{ or } n = -3$$
$$n = 6; \text{ reject } n = -3.$$

36. $D = \dfrac{n(n-3)}{2}$

$n = \dfrac{n(n-3)}{2}$

$2n = n^2 - 3n$

$0 = n^2 - 5n$

$0 = n(n-5)$

$n = 0$ or $n - 5 = 0$

$n = 0$ or $\qquad n = 5$

$n = 5$; reject $n = 0$

37. $c^2 = a^2 + b^2$

$c^2 = 3^2 + 4^2$

$c^2 = 9 + 16$

$c^2 = 25$

$c = \pm 5$

$c = 5$; reject $c = -5$

38. $c^2 = a^2 + b^2$

$10^2 = 6^2 + b^2$

$100 - 36 = b^2$

$64 = b^2$

$b = \pm 8$

$b = 8$; reject $b = -8$

39. $a^2 + (a+3)^2 = (a+4)^2$

$a^2 + a^2 + 6a + 9 = a^2 + 8a + 16$

$a^2 - 2a - 7 = 0$

Use $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{2 \pm \sqrt{4 - 4(1)(-7)}}{2 \cdot 1}$

$x = \dfrac{2 \pm \sqrt{4 + 28}}{2}$

$x = \dfrac{2 \pm \sqrt{32}}{2}$

$x = \dfrac{2 \pm 4\sqrt{2}}{2}$

$x = 1 + 2\sqrt{2}$ or $x = 1 - 2\sqrt{2}$

Reject the negative value, $1 - 2\sqrt{2}$.

$\therefore\ a = 1 + 2\sqrt{2}$